STATE OF STRESS IN THE NEIGHBORHOOD OF A ROUGH SURFACE OF ELASTIC BODIES

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It is known that the surface of machine parts is rough. The roughness changes the state of stress in the neighborhood of the surface as compared with the case of an ideal smooth surface.

The present paper is devoted to the clarification of the question of the magnitude of these changes. The rough surface is considered as the realization of a homogeneous statistically anisotropic random field with a normal law of distribution.

It is assumed that the material of the elastic body is isotopic.

1. We consider an elastic semi-space $z \geqslant H(x, y)$, subjected at infinity $z \to \infty$ to the action of normal stresses σ_1 and σ_2 (Fig. 1).

We assume that the boundary of the semi-space $z = H(x, y)$ differs little from the plane $z = 0$. Moreover, we assume the boundary stress-free

$$\gamma_1\sigma_x + \gamma_2\tau_{xy} + \gamma_3\tau_{xz} = 0, \qquad \gamma_1\tau_{xy} + \gamma_2\sigma_y + \gamma_3\tau_{yz} = 0 \qquad (1.1)$$
$$\gamma_1\tau_{xz} + \gamma_2\tau_{yz} + \gamma_3\sigma_z = 0 \quad \text{for } z = H(x, y)$$

Here σ_x, σ_y, τ_{xy} etc. are the components of the stress tensor; and γ_1, γ_2 and γ_3 are the direction cosines of the outer normal to the surface $z = H(x, y)$. The latter, with an accuracy up to the magnitudes of first order with respect to H are

$$\gamma_1 = \frac{\partial H}{\partial x}, \qquad \gamma_2 = \frac{\partial H}{\partial y}, \qquad \gamma_3 = -1 \qquad (1.2)$$

We write the expressions for the stresses at $z = H(x, y)$ by means of expansions

$$\sigma_x|_{z=H} = \sigma_x|_{z=0} + H \frac{\partial \sigma_x}{\partial z}\Big|_{z=0} + \dots \qquad \text{etc.} \tag{1.3}$$

We substitute (1.3) and (1.2) into (1.1) and write the boundary conditions on the free surface with an accuracy up to the magnitudes of first order with respect to H and its derivatives

$$\begin{aligned} \tau_{zx} &= \sigma_x \frac{\partial H}{\partial x} + \tau_{xy} \frac{\partial H}{\partial y} - H \frac{\partial \tau_{zx}}{\partial z} \\ \tau_{zy} &= \tau_{xy} \frac{\partial H}{\partial x} + \sigma_y \frac{\partial H}{\partial y} - H \frac{\partial \tau_{zy}}{\partial z} \qquad \text{for } z = 0 \\ \sigma_z &= \tau_{zx} \frac{\partial H}{\partial x} + \tau_{zy} \frac{\partial H}{\partial y} - H \frac{\partial \sigma_z}{\partial z} \end{aligned} \tag{1.4}$$

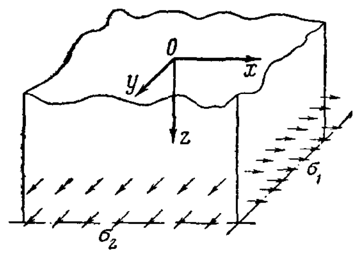

Fig. 1.

The stresses we are seeking must satisfy the boundary conditions (1.4), the loading conditions at infinity ($z \to \infty$) and the entire system of equations in terms of stresses of the theory of elasticity. We find the solution of the problems by the method of successive approximations, taking as the first approximation the solution of the problem for $H = 0$. Obviously in this case we have in the entire region occupied by the body

$$\sigma_x = \sigma_1, \quad \sigma_y = \sigma_2, \quad \sigma_z = 0; \qquad \tau_{xy} = 0, \quad \tau_{xz} = 0, \quad \tau_{yz} = 0 \tag{1.5}$$

We take the second approximation in the form

$$\begin{aligned} \sigma_x &= \sigma_1 + \sigma_x^{(1)}, \qquad \sigma_y = \sigma_2 + \sigma_y^{(1)}, \qquad \sigma_z = \sigma_z^{(1)} \\ \tau_{xy} &= \tau_{xy}^{(1)}, \qquad \tau_{xz} = \tau_{xz}^{(1)}, \qquad \tau_{yz} = \tau_{yz}^{(1)} \end{aligned} \tag{1.6}$$

For determination of stresses $\sigma_x^{(1)}$ and $\sigma_y^{(1)}$ etc., we have the boundary conditions

$$\tau_{xz}^{(1)} = \sigma_1 \frac{\partial H}{\partial x}, \qquad \tau_{yz}^{(1)} = \sigma_2 \frac{\partial H}{\partial y}, \qquad \sigma_z^{(1)} = 0 \qquad \text{for } z = 0 \tag{1.7}$$

which are obtained by substituting the stresses of the first approximation on the right-hand sides of (1.4); we have also the condition of $\sigma_x^{(1)}$, $\sigma_y^{(1)}$ etc. becoming zero for $z \to \infty$. These conditions correspond to the problem of the action of tangential stresses on a semi-space. In [1] its solution is given in the form

$$\sigma_x^{(1)} = 2\left[\frac{\partial^2 \psi_1}{\partial z \partial x} - \frac{z}{2} \frac{\partial^2}{\partial x^2}\left(\frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}\right) + \frac{1}{m} \int_z^\infty \frac{\partial^2}{\partial y^2}\left(\frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y}\right) dz\right]$$

$$\sigma_y^{(1)} = 2\left[\frac{\partial^2\psi_2}{\partial z\partial y} - \frac{z}{2}\frac{\partial^2}{\partial y^2}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right) + \frac{1}{m}\int_z^\infty \frac{\partial^2}{\partial x^2}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right) dz\right]$$

$$\tau_{xy}^{(1)} = \frac{\partial}{\partial z}\left(\frac{\partial\psi_1}{\partial y} + \frac{\partial\psi_2}{\partial x}\right) - z\frac{\partial^2}{\partial x\partial y}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right) - \frac{2}{m}\int_z^\infty \frac{\partial^2}{\partial x\partial y}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right)\partial z$$

$$\tau_{xz}^{(1)} = \frac{\partial^2\psi_1}{\partial z^2} - z\frac{\partial^2}{\partial z\partial x}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right), \qquad \tau_{yz}^{(1)} = \frac{\partial^2\psi_2}{\partial z^2} - z\frac{\partial^2}{\partial z\partial y}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right)$$

$$\sigma_z^{(1)} = -z\frac{\partial^2}{\partial z^2}\left(\frac{\partial\psi_1}{\partial x} + \frac{\partial\psi_2}{\partial y}\right) \tag{1.8}$$

Here m is Poisson's ratio, and ψ_1 and ψ_2 are harmonic functions in the semi-space $z > 0$, tending to zero for $z \to \infty$, and satisfying the boundary conditions for $z = 0$

$$\frac{\partial^2\psi_1}{\partial z^2} = \tau_{zx}^{(1)} = \sigma_1\frac{\partial H}{\partial x}, \qquad \frac{\partial^2\psi_2}{\partial z^2} = \tau_{zy}^{(1)} = \sigma_2\frac{\partial H}{\partial y} \tag{1.9}$$

Assume that the stresses in the second approximation are found. However, not one of them, separately, fully characterizes the state of stress. But in computing the coefficient of the stress concentration it is desirable to introduce some single invariant characteristic of the state of stress. Such a characteristic, at least for plastic materials, is the intensity of shear stresses T, which is determined by the following formula (see, for example, [2]):

$$T = \frac{1}{\sqrt{6}}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y + \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\,(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{xz}^2)\right]^{1/2} \tag{1.10}$$

We substitute into (1.10) the expressions for the stresses from (1.6) and, assuming the disturbances $\sigma_x^{(1)}$, $\sigma_y^{(1)}$ etc. to be small, we linearize the result. In the formula obtained

$$T = T_0 + \frac{1}{6T_0}\left[(2\sigma_1 - \sigma_2)\,\sigma_x^{(1)} + (2\sigma_2 - \sigma_1)\,\sigma_y^{(1)}\right] \tag{1.11}$$

T_0 represents the value of the intensity of shear stresses in the absence of disturbances

$$T_0 = \frac{1}{\sqrt{3}}\,(\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2)^{1/2} \tag{1.12}$$

2. Assume that the surface $z = H(x, y)$ represents a random homogeneous anisotropic field (see [3]) with zero value of the mathematical expectation. The correlation function of such a field

$$K\,(x - x_1, y - y_1) = M\,[H^*(x, y)\,H(x_1, y_1)] \tag{2.1}$$

depends only on the differences $x - x_1$ and $y - y_1$, and the field itself admits a spectral decomposition

$$H(x, y) = \iint\limits_{-\infty}^{\infty} e^{i(\mu x + \nu y)} V(\mu, \nu)\, d\mu\, d\nu \tag{2.2}$$

Here $V(\mu, \nu)$ is a random function of the arguments μ and ν with zero value of the mathematical expectation and the correlation function

$$M\,[V^*(\mu, \nu) V(\mu_1, \nu_1)] = S(\mu, \nu)\, \delta(\mu - \mu_1)\, \delta(\nu - \nu_1) \tag{2.3}$$

where $M[\ldots]$ denotes the operation of mathematical expectation, δ is the delta function, and the star denotes the complex-conjugate magnitude. The nonrandom function $S(\mu, \nu)$ is called the spectral density of the homogeneous random field $H(x, y)$.

Substituting H from (2.2) into (1.9), we obtain the boundary conditions

$$\frac{\partial^2 \psi_1}{\partial z^2} = \sigma_1 \iint\limits_{-\infty}^{\infty} i\mu e^{i(\mu x + \nu y)} V d\mu d\nu, \qquad \frac{\partial^2 \psi_2}{\partial z^2} = \sigma_2 \iint\limits_{-\infty}^{\infty} i\nu e^{i(\mu x + \nu y)} V d\mu d\nu \quad \text{when } z = 0 \tag{2.4}$$

With their aid one finds easily the harmonic functions in the semi-space $z > 0$

$$\begin{aligned} \psi_1 &= \iint\limits_{-\infty}^{\infty} \frac{i\mu\sigma_1 V}{\mu^2 + \nu^2} \exp\left[- z \sqrt{\mu^2 + \nu^2} + i(\mu x + \nu y)\right] d\mu\, d\nu \\ \psi_2 &= \iint\limits_{-\infty}^{\infty} \frac{i\nu\sigma_2 V}{\mu^2 + \nu^2} \exp\left[- z \sqrt{\mu^2 + \nu^2} + i(\mu x + \nu y)\right] d\mu d\nu \end{aligned} \tag{2.5}$$

In investigating the stress concentration, the greatest interest lies in the knowledge of the stresses on the boundary of the semi-space. The latter coincide with an accuracy up to magnitudes of first order, with the values of the same stresses for $z = 0$. Therefore, we give the expressions for $\sigma_x^{(1)}$ and $\sigma_x^{(2)}$ for $z = 0$

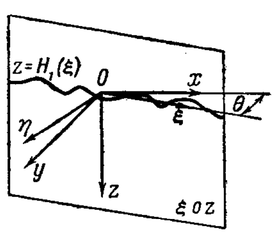

Fig. 2.

$$\sigma_x^{(1)} = 2 \iint\limits_{-\infty}^{\infty} \left[\mu^2\sigma_1 + \frac{\nu^2}{m} \frac{\mu^2\sigma_1 + \nu^2\sigma_2}{\mu + \nu^2}\right] \frac{e^{i(\mu x + \nu y)}}{\sqrt{\mu^2 + \nu^2}} V d\mu d\nu$$

$$\sigma_y^{(1)} = 2 \iint\limits_{-\infty}^{\infty} \left[\nu^2\sigma_2 + \frac{\mu^2}{m} \frac{\mu^2\sigma_1 + \nu^2\sigma_2}{\mu^2 + \nu^2}\right] \frac{e^{i(\mu x + \nu y)}}{\sqrt{\mu^2 + \nu^2}} V d\mu d\nu$$

etc. (2.6)

Introducing the expressions found for the disturbances into (1.11), we obtain the value of the intensity of the shear stresses on the surface of the elastic body

$$T = T_0 + \frac{1}{3T_0} \iint_{-\infty}^{\infty} \Phi(\mu, \nu)\, e^{i(\mu x + \nu y)} V d\mu d\nu \tag{2.7}$$

The function φ has the following form:

$$\Phi(\mu, \nu) = \frac{2\mu^2\sigma_1^2 + 2\nu^2\sigma_2^2 - (\mu^2 + \nu^2)\,\sigma_1\sigma_2}{(\mu^2 + \nu^2)^{1/2}} + \\ + \frac{1}{m}\,\frac{(\mu^2\sigma_1 + \nu^2\sigma_2)\,[(2\sigma_1 - \sigma_2)\,\nu^2 + (2\sigma_2 - \sigma_1)\,\mu^2]}{(\mu^2 + \nu^2)^{3/2}} \tag{2.8}$$

The mathematical expectation and the dispersion T are written with the aid of formulas (2.7) and (2.3) as

$$M\,[T] = T_0, \qquad D = \frac{1}{9T_0^2} \iint_{-\infty}^{\infty} \Phi^2(\mu, \nu)\, S(\mu, \nu)\, d\mu d\nu \tag{2.9}$$

As the coefficient of stress concentration we take the magnitude

$$\alpha = 1 + 2\,\frac{D^{1/2}}{T_0} \tag{2.10}$$

3. Let the surface $z = H(x, y)$ be generated by a translation motion of the curve

$$z = H_1(\xi) \tag{3.1}$$

along the η-axis. The position of the η- and ξ-axes is shown in Fig. 2.

The cylindrical surface obtained is a sufficiently good idealization of the real surface of machine parts after machining (shaping, milling and grinding).

We assume that the function $H_1(\xi)$ is the so-called stationary random function of the argument ξ. We denote its correlation function by $K_1(\xi - \xi_1)$, and the spectral density by $S_1(\omega)$. Between them there exists the known relation (see [3])

$$S_1(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} K_1(\xi)\, e^{-i\omega\xi} d\xi, \qquad K_1(\xi) = \int_{-\infty}^{\infty} S_1(\omega)\, e^{i\omega\xi} d\omega \tag{3.2}$$

The surface $z = H(x, y)$ is cylindrical, therefore the correlation function of the random field $H(x, y)$ has the form

$$K(x - x_1, y - y_1) = K_1(\xi - \xi_1) \tag{3.3}$$

whereby ξ, η and x, y are connected by the relations

$$\xi = x\cos\theta + y\sin\theta, \qquad \eta = y\cos\theta - x\sin\theta \tag{3.4}$$

$$x = \xi\cos\theta - \eta\sin\theta, \qquad y = \xi\sin\theta + \eta\cos\theta \tag{3.5}$$

Let us find the spectral density of the random field $H(x, y)$. By definition [3]

$$S(\mu, \nu) = \frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty} K(x, y)\, e^{-i(\mu x + \nu y)}\,dx\,dy \tag{3.6}$$

We substitute here the expression for the correlation function from (3.3) and integrate with respect to the variables ξ and η

$$S(\mu, \nu) = \frac{1}{2\pi}\int_{-\infty}^{\infty} K_1(\xi)\, e^{-i\xi(\mu\cos\theta + \nu\sin\theta)}\,d\xi\, \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-i\eta(\nu\cos\theta - \mu\sin\theta)}\,d\eta \tag{3.7}$$

The first integral by virtue of (3.2) is

$$S_1(\mu\cos\theta + \nu\sin\theta)$$

the second integral is the delta function

$$\delta(\nu\cos\theta - \mu\sin\theta)$$

Thus we obtain

$$S(\mu, \nu) = S_1(\mu\cos\theta + \nu\sin\theta)\,\delta(\nu\cos\theta - \mu\sin\theta) \tag{3.8}$$

With the aid of (3.8) we calculate according to (2.9) the dispersion T; we have

$$D = \frac{1}{9T_0^2}\iint_{-\infty}^{\infty} \Phi^2(\mu, \nu)\, S_1(\mu\cos\theta + \nu\sin\theta)\,\delta(\nu\cos\theta - \mu\sin\theta)\,d\mu\,d\nu \tag{3.9}$$

Integrating with respect to ν, we obtain

$$D = \frac{1}{9T_0^2}\int_{-\infty}^{\infty} \Phi^2(\mu, \mu\tan\theta)\, S_1\left(\frac{\mu}{\cos\theta}\right)\frac{d\mu}{\cos\theta} \tag{3.10}$$

Introducing a new variable ω by means of the relation $\mu = \omega\cos\theta$, we obtain

$$D = \frac{1}{9T_0^2}\int_{-\infty}^{\infty} \Phi^2(\omega\cos\theta, \omega\sin\theta)\, S_1(\omega)\,d\omega \tag{3.11}$$

Since $\varphi(\mu, \nu)$ is a homogeneous function of first degree with respect to its argument, and at the same time an even function, the relation

$$\Phi(\omega\cos\theta, \omega\sin\theta) = |\omega|\,\Phi(\cos\theta, \sin\theta) \tag{3.12}$$

is true; this permits us to write D in the form

$$D = \frac{\Phi^2(\cos\theta, \sin\theta)}{9T_0^2} \int_{-\infty}^{\infty} \omega^2 S_1(\omega)\, d\omega \tag{3.13}$$

The integral, by virtue of the second formula (3.2), is expressed through the derivative of the correlation function thus

$$\int_{-\infty}^{\infty} \omega^2 S_1(\omega)\, d\omega = -\left.\frac{d^2K_1(\xi)}{d\xi^2}\right|_{\xi=0} \tag{3.14}$$

We eliminate the derivative K_1 by means of the formula (see [4])

$$n = \frac{1}{\pi}\left[-\frac{1}{K_1(0)}\left.\frac{d^2K_1(\xi)}{d\xi^2}\right|_{\xi=0}\right]^{1/2} \tag{3.15}$$

which gives the mathematical expectation of the mean number of intersections of the zero level of the normal random function $H_1(\xi)$. Denoting

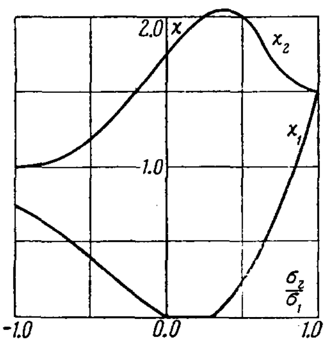

Fig. 3.

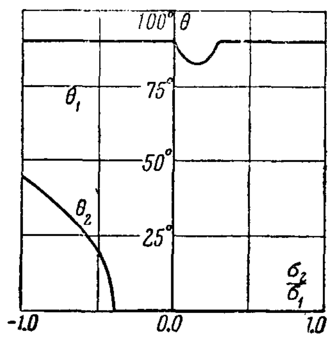

Fig. 4.

by h the mean square value of the function $H_1(\xi)$, we find

$$\int_{-\infty}^{\infty} \omega^2 S_1(\omega)\, d\omega = (\pi n h)^2 \tag{3.16}$$

Substituting (3.16) into (3.13) we obtain

$$D = \frac{\Phi^2(\cos\theta, \sin\theta)}{9T_0^2} (\pi n h)^2 \tag{3.17}$$

Then we determine the mean square value T

$$D^{1/2} = \pi n h T_0 \chi \tag{3.17}$$

where

$$\chi = \frac{|\Phi(\cos\theta, \sin\theta)|}{3T_0^2} = \frac{1}{\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2}\Big| 2\sigma_1^2 \cos^2\theta + 2\sigma_2^2 \sin^2\theta - \sigma_1\sigma_2 + $$
$$+ \frac{\sigma_1 \cos^2\theta + \sigma_2 \sin^2\theta}{m} [(2\sigma_1 - \sigma_2)\sin^2\theta + (2\sigma_2 - \sigma_1)\cos^2\theta]\Big| \qquad (3.19)$$

The expression for the concentration coefficient is found by (2.10)

$$\alpha = 1 + 2\pi nh\chi \qquad (3.20)$$

Since the concentration coefficient depends on θ, it is possible to diminish it by selecting θ, and consequently the direction of machining. From (3.19) it follows that to each value of the ratio σ_2/σ_1 there corresponds a definite value $\theta = \theta_1$ on the interval $[0, 90^\circ]$, for which the minimum $\chi = \chi_1$ is obtained, and therefore the minimum of the concentrated coefficient. The relation $\chi = \chi_1(\sigma_2/\sigma_1)$ and $\theta = \theta_1(\sigma_2/\sigma_1)$ in the assumption $|\sigma_2| \leqslant |\sigma_1|$, $m = 4$ are shown in Figs. 3 and 4. There are also shown, for comparison, the relations $\chi = \chi_2(\sigma_2/\sigma_1)$ and $\theta = \theta_2(\sigma_2/\sigma_1)$ corresponding to the maximum value of the concentration coefficient. From Fig. 4 follows an approximate conclusion: for given σ_1 and σ_2 the minimum value of χ, and consequently of the concentration coefficient, is obtained when the generatrix of the cylindrical surface $z = H(x, y)$ coincides with the direction of the maximum value of the stress (σ_1 or σ_2). This means that minimum α is obtained when the direction of the machining coincides with the line of action of the maximal principal stress in the undisturbed stress field.

We give expressions for the concentration coefficient of stresses for some special cases of undisturbed state of stress and orientation of the direction of machining.

a) All-sided uniform tension ($\sigma_1 = \sigma_2$)

$$\alpha = 1 + 2\pi nh \left(1 + \frac{2}{m}\right) \qquad (3.21)$$

b) Uniaxial tension ($\sigma_2 = 0$); direction of machining perpendicular to the line of action of stress $\sigma_1(\theta = 0)$

$$\alpha = 1 + 2\pi nh \left(2 - \frac{1}{m}\right) \qquad (3.22)$$

c) State of pure shear; direction of machining makes an angle of 45° with the line of action of one of the principal stresses

$$\alpha = 1 + 2\pi nh \qquad (3.23)$$

4. Let the homogeneous random field $H(x, y)$ be statistically isotropic. It is known [3] that the correlation function of such a field depends only on the distance between the points of observation r, and the spectral density depends only on the magnitude ω

$$K(x, y) = (r), \quad r = \sqrt{x^2 + y^2}, \qquad S(\mu, \nu) = S(\omega), \qquad \omega = \sqrt{\mu^2 + \nu^2} \tag{4.1}$$

In the case of such a form of the spectral density the expression for D by (2.9) may be substantially simplified.

Substitute S from (4.1) into (2.9). In calculating the integral obtained, it is expedient to use polar coordinates, setting

$$\mu = \omega \cos \varphi, \qquad \nu = \omega \sin \varphi \tag{4.2}$$

and using property (3.12) of the function $\varphi(\mu, \nu)$. Then D becomes

$$D = \frac{1}{9T_0^2} \int_0^\infty \omega^3 S(\omega)\, d\omega \int_0^{2\pi} \Phi^2 (\cos \varphi, \sin \varphi)\, d\varphi \tag{4.3}$$

The second integral is easily calculated if one uses the representation

$$\Phi^2 (\cos \varphi, \sin \varphi) = b_0 \sin^8 \varphi + b_1 \sin^6 \varphi \cos^2 \varphi + + b_2 \sin^4 \varphi \cos^4 \varphi + b_3 \sin^2 \varphi \cos^8 \varphi + b_4 \cos^8 \varphi \tag{4.4}$$

and applies the integral

$$\int_0^{1/2\pi} \sin^{\beta-1} \varphi \cos^{\gamma-1} \varphi d\varphi = \frac{1}{2} \mathrm{B} \left(\frac{\beta}{2}, \frac{\gamma}{2}\right) \tag{4.5}$$

Here B is the Euler beta function.

The final expression of the second integral in (4.3) is

$$\int_0^{2\pi} \Phi^2 (\cos \varphi, \sin \varphi)\, d\varphi = \frac{\pi}{8^2} (35 b_0 + 5 b_1 + 3 b_2 + 5 b_3 + 35\, b_4) \tag{4.6}$$

The coefficients b_i have the form

$$b_0 = A^2, \qquad b_1 = 4\left(1 + \frac{1}{m}\right) AC, \qquad b_2 = 4\left(1 + \frac{1}{m}\right)^2 c^2 + 2AB$$

$$b_3 = 4\left(1 + \frac{1}{m}\right) BC, \qquad b_4 = B^2 \tag{4.7}$$

where

$$A = \sigma_2\left(2\sigma_2 - \sigma_1 + \frac{2\sigma_1 - \sigma_2}{m}\right), \quad B = \sigma_1\left(2\sigma_1 - \sigma_2 + \frac{2\sigma_2 - \sigma_1}{m}\right)$$
$$C = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \qquad (4.8)$$

The first integral in (4.3) gives a simple expression in directly measurable statistical characteristics of the profile of the rough surface. To show this we write the correlation function of the field

$$K(x, y) = \iint_{-\infty}^{\infty} e^{i(\mu x + \nu y)} S(\mu, \nu)\, d\mu\, d\nu \qquad (4.9)$$

Substituting for the spectral density of the field from (4.1), and introducing polar coordinates (4.2) and $x = r\cos\psi$, $y = r\sin\psi$, we find

$$K(x, y) = K(r) = \int_0^\infty S(\omega)\,\omega d\omega \int_0^{2\pi} e^{i\omega r\cos(\varphi - \psi)}\, d\varphi \qquad (4.10)$$

The second derivative of the correlation function $K(r)$ for $r = 0$ enters into (4.3)

$$K''(0) = -\pi \int_0^\infty \omega^3 S(\omega)\, d\omega \qquad (4.11)$$

Noting that the function $K(r)$ coincides with the correlation function of the ordinates of the profile of the rough surface, we eliminate it from (4.11) by means of a formula analogous to (3.15). As a result we obtain

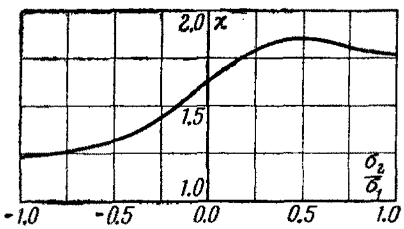

Fig. 5.

$$\int_0^\infty \omega^3 S(\omega)\, d\omega = \frac{1}{\pi}(\pi n h)^2 \qquad (4.12)$$

Here h is the mean square value of the field $H(x, y)$, and n is the mathematical expectation of the mean number of intersections of the zero level by the profile of the rough surface. According to the above formula (4.12), as well as (3.15), this holds true for a random field H with a normal law of distribution.

Introducing the integrals (4.12) and (4.6) into (4.3), we find

$$D = (\pi n h T_0 \chi)^2, \qquad \chi = \frac{1}{24 T_0^2}(35 b_0 + 5 b_1 + 3 b_2 + 5 b_3 + 35 b_4)^{1/2} \qquad (4.13)$$

which, in agreement with (2.10), gives the following expression for the concentration coefficient of the stresses:

$$\alpha = 1 + 2\pi nh\chi \qquad (4.14)$$

The result of computing χ for $m = 4$ and various values of the ratio σ_2/σ_1 is given in Fig. 5.

In conclusion we remark that the formulas (3.21), (3.22), (3.23), (3.20) and (4.14) are convenient in practical calculations, since they contain easily measurable statistical characteristics of the profile of the rough surface.

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